An Algebraic Algorithm for the Production-Inventory-Transportation Problem in a Three-Stage Supply

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AN ALGEBRAIC ALGORITHM FOR THE INTEGRATED INVENTORY-TRANSPORTATION SUPPLY CHAIN PROBLEM

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Abstract:

In this paper, we develop a three-stage, serial supply chain production inventory model with the integration of transportation cost. This supply chain model is formulated for the integer multipliers coordination mechanism, where firms at the same stage of the supply chain use the same cycle time and the cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage. We develop an optimal replenishment policy using a simple algebraic procedure to solve the problem without the use of differential calculus.

Keywords: Algebraic Algorithm, Supply Chain Management, Inventory Model.

1. INTRODUCTION

The benefits of inventory coordination and information sharing among the supply chain participants, have received significant attention in the literature. Research findings in this area revealed that information sharing and coordinated inventory replenishments can help reduce the inventory and order costs as well as transportation costs. Collaborative transportation management in supply chain can significantly reduce the retailer’s total costs and improve the retailer’s service level [5].

The transportation cost is explicitly incorporated into the integrated vendor–buyer supply chain model by [4]. Then, optimal solution procedures considering all-unit-discount transportation cost structures is developed. A model for a two-stage supply chain consisting of two distributors and two retailers is developed by [1] along with a decision rule that minimizes the total expected cost associated with all outstanding orders at the time of order placement. The case of the single-supplier multi-retailers supply chain is considered by [6], where they developed heuristic algorithms by simultaneously considering inventory and transportation decisions.

In this paper, we extend the model in [4] to consider a three-stage supply chain model and use a different and easier approach to develop the solution procedure. The remainder of this paper is organized as follows. The next section presents the problem definition, assumptions and notations. Section 3 describes the development of the model for the case the transportation cost not considered. The integration of the transportation cost and the details of the solution algorithm are presented in section 4. A numerical example is given in section 5. Finally, section 6 contains some concluding remarks.

2. NOTATION AND ASSUMPTIONS

As mentioned in section one, [4] introduced the concept of explicit integration of the transportation cost into the joint vendor–buyer supply chain model. They incorporated the transportation cost explicitly into the model and developed optimal solution procedures for solving the integrated models. But this model only considered the two stage supply chain model. The following notations are used in developing the extended three-stage supply model:

T =Basic cycle time, cycle time at the end retailer
T_i =Cycle time at stage i
T_s = The duration of the stock-out period
S_i =Setup cost at stage i
K_i =Integer multiplier at stage i
h_i =Inventory holding cost at stage i
D =The demand rate at the end stage
P_i =Production rate at stage i

Assumptions for the multi-stage supply chain production-inventory model:
A single product is produced and distributed through a four-stage serial, supply chain.

Replenishment is instantaneous.

Production rates and Demand rate are deterministic and uniform.

A lot produced at stage is sent in equal shipments to the downstream stage.

The entire supply chain optimization is acceptable for all partners in the chain.

Complete information sharing policy is adopted.

Cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage.

Shortage is not allowed.

3. THE MODEL WITHOUT TRANSPORTATION COSTS CONSIDERATION

We consider a three-stage, serial supply chain. This supply chain model is formulated for the integer multipliers coordination mechanism, where the cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage. In this case, the time-weighted total cost for the retailer is given by:

$$ TC_3 = h_3 \frac{TD}{2} + \frac{S_3}{T} $$

The holding cost at each stage, except for the final stage (the end retailer’s stage 3), is made of two parts: the first one is the carrying cost for the raw materials as they are being converted into finished products during the production portion of the cycle. The second part is the carrying cost of the finished products during the non-production portion of the cycle as in [7]. Therefore, the total time-weighted cost for each of the second and first stages is represented by following cost equations respectively:

$$ TC_2 = K_2 \frac{TD^2}{2P_2} (h_1 + h_2) + \left( K_2 - 1 \right) \frac{TD}{2} h_2 + \frac{S_2}{K_2 T} $$

$$ TC_1 = K_1 K_2 \frac{TD^2}{2P_1} (h_0 + h_1) + K_2 \left( K_1 - 1 \right) \frac{TD}{2} h_1 + \frac{S_2}{K_1 K_2 T} $$

The entire supply time-weighted cost is

$$ TC = \sum TC_i = $$

Rewriting equation (4), one has

$$ TC = TY + \frac{W}{T} $$

Where

$$ Y = \frac{K_2 \psi_2 + \alpha_2}{2} $$

$$ \alpha_2 = Dh_3 - Dh_2 $$

$$ \psi_2 = K_1 \psi_1 + \alpha_1 $$

$$ \alpha_1 = \frac{D^2}{P_2} (h_1 + h_2) + Dh_2 - Dh_1 $$

$$ \psi_1 = \frac{D^2}{P_1} (h_0 + h_1) + Dh_1 $$

Now applying the algebraic procedure proposed by [2] and [3], the annual total cost for the entire supply chain in Eq. (5) can be represented by factorizing the term $1/T$ and completing the perfect square, one has

$$ TC = \frac{1}{T} \left( T^2 Y - 2T \sqrt{YW} + W + 2T \sqrt{YW} \right) $$

Factorizing the perfect squared trinomial in a squared binomial we obtain:

$$ TC = \frac{1}{T} \left( T \sqrt{Y} - \sqrt{W} \right)^2 + 2T \sqrt{YW} $$

It is worthy pointing out that Eq. (7) reaches minimum with respect to $T$ when setting

$$ \left( T \sqrt{Y} - \sqrt{W} \right)^2 = 0 $$

Hence, the optimal basic cycle time $T^*$ is

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Substituting Eq (8) into Eq.(7), the minimum value for the annual total cost for the entire supply chain minimum cost is

\[ TC = 2\sqrt{YW} \]  

(9)

The optimal basic cycle time \( T* \) is a function of the integer multipliers \( (K_2, K_1) \). We use the method of perfect square to drive the optimal values of these integer multipliers iteratively. Substituting for \( Y \) and \( W \) into Eq.(9) we get

\[ \sqrt{\left(\frac{1}{K_2} \left[ \sqrt{\psi_2 S_3} - \sqrt{\alpha_2 \varphi_2} \right] + \sqrt{\psi_2 \varphi_2} + \sqrt{\alpha_2 S_2} \right)^2} = \]

(10)

From (10) setting

\[ \left[ K_2 \sqrt{\psi_2 S_3} - \sqrt{\alpha_2 \varphi_2} \right]^2 = 0 \]

the optimal value of integer multiplier \( K_2 \) is derived as follows:

\[ K_2^* = \frac{\sqrt{\alpha_2 \varphi_2}}{\sqrt{\psi_2 S_3}} \]  

(11)

Since the value of \( K_2 \) is a positive integer, the following condition must be satisfied:

\[ (K_2^* - 1) \leq (K_2^*)^2 \leq K_2^* + 1 \]

To drive the optimal value of integer multiplier \( K_1 \), we can rewrite the term \( \sqrt{\psi_2 \varphi_2} \) in Eq. (10) as follows:

\[ \sqrt{\psi_2 \varphi_2} = \left\{ \frac{1}{K_1} \left[ \sqrt{\psi_1 S_2} - \sqrt{\alpha_1 \varphi_1} \right]^2 + \sqrt{\psi_1 \varphi_1} + \sqrt{\alpha_1 S_1} \right\} \]  

(12)

From Eq.(14), setting

\[ \left[ K_1 \sqrt{\psi_1 S_2} - \sqrt{\alpha_1 \varphi_1} \right]^2 = 0 \]

And the optimal value of the integer multiplier \( K_1 \) is derived as follows:

\[ K_1^* = \frac{\sqrt{\alpha_1 S_1}}{\sqrt{\psi_1 S_2}} = \left[ \frac{D(h_0 + h_1)}{P_1 + h_1 - h_0} \right]^{1/2} \]  

(13)

Since the value of \( K_1 \) is a positive integer, the following condition must be satisfied:

\[ (K_1^* - 1) \leq (K_1^*)^2 \leq K_1^* + 1 \]

Now, we can use \( K_1^* \) to find \( K_2^* \) and \( T^* \).

4. THE MODEL WITH THE TRANSPORTATION COST

In the previous section, the supply chain production inventory model was presented without consideration of the transportation cost. In this section we integrate the transportation cost to the model and develop the solution algorithm. The transportation cost consists of two parts: the cost of transporting the products from the first stage to the second stage \( TT_{12} \), and the cost of transporting these products from the second stage to the third stage \( TT_{23} \). Considering these two costs, the expected total cost per unit time can be expressed as:

\[ TC = TY + \frac{W}{T} + TT_{12} + TT_{23} \]  

(14)

The quantity discount transportation cost structure followed in the development of the model is defined as:

\[ TT_{i,j} = \begin{cases} c_0 D & Q \in [0, M_1] \\ c_1 D & Q \in [M_1, M_2] \\ \vdots & \vdots & \vdots \\ c_m D & Q \in [M_m, \infty] \end{cases} \]

Where \( Q \) is transported quantity and \( c_0 < \ldots < c_1 < \ldots c_m \) are the different unit transportation costs as in [4].

Now, Eq.(16) represents the integrated production inventory transportation supply model with planned backorders. The model in [4] considered a similar supply chain structure but made up of two stages only. The authors of [4] identified some properties of the solution and developed a solution algorithm. In the following we use the algebraic approach detailed above to extend their algorithm to solve a three-stage supply chain model. The solution algorithm starts with the determination of the optimal integer multipliers first. Then, it performs some search to find the optimal shipment size considering the transportation costs.

The Solution Algorithm

1. Use Equation(13) to find \( K_1^* \).
2. Use Equation (11) to find \( K_2^* \).
3. Use Equation (8) to find \( T^* \).
4. If \( M_m < K_2^* T^* D \) then stop this is the optimal solution. Otherwise go to step 6.
5. For each \( K_1 = 1, 2, \ldots K_1^* \)
   5.1 Find \( K_2 \) using Equation (11).
   5.2 Find \( T \) using Equation (8).
5.3 Calculate the total cost using Equation (16).

6. The optimal solution is one resulting in the minimum cost in step 5.4.

5. NUMERICAL EXAMPLE

In this section, we consider an example of a three-stage supply chain consisting of a supplier, a manufacturer, a distributor, and a retailer. The relevant data is shown in Table 1. In addition to this data, it is also assumed that holding cost for the supplier’s supplier is \( h_0 = 0.5 \).

Table 1: Data for the Example Supply Chain

<table>
<thead>
<tr>
<th>I</th>
<th>P_i</th>
<th>D</th>
<th>h_i</th>
<th>S_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4000</td>
<td>13000</td>
<td>0.8</td>
<td>700</td>
</tr>
<tr>
<td>2</td>
<td>30000</td>
<td>13000</td>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>30000</td>
<td>13000</td>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

The following all units discount transportation cost scheme is used in this example:

\[
\begin{align*}
Q < 2000 & \quad 0.35 \\
2000 \leq Q < 2500 & \quad 0.25 \\
2500 \leq Q < 3000 & \quad 0.20 \\
3000 \leq Q < 4500 & \quad 0.18 \\
4500 \leq Q < 6000 & \quad 0.15 \\
6000 \leq Q < 7500 & \quad 0.10 \\
7500 \leq Q & \quad 0.05
\end{align*}
\]

By applying the developed algebraic solution procedure in section 4, the results of this example are presented in Tables 2 and 3.

Table 2: The Example Solution without Transportation Cost Consideration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 )</td>
<td>3</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>2</td>
</tr>
<tr>
<td>( T )</td>
<td>0.00842</td>
</tr>
<tr>
<td>( TC )</td>
<td>16223.845</td>
</tr>
</tbody>
</table>

Table 3: The Example Solution with Transportation Cost Consideration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 )</td>
<td>1</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>2</td>
</tr>
<tr>
<td>( T )</td>
<td>0.014465884</td>
</tr>
</tbody>
</table>

6. CONCLUSION

In this study, we developed a three-stage, serial supply chain inventory model with transportation cost. The problem is to coordinate production and inventory decisions across the supply chain so that the total cost of the system is minimized. This supply chain model is formulated for the integer multipliers coordination mechanism, where firms at the same stage of the supply chain use the same cycle time and the cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage. We developed an optimal replenishment and transportation algorithm using a simple algebraic method. Future research can investigate the extension of the model to have four stages or more.

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References:


